equal. Dilatation effects scale roughly with M_e^2; at higher Mach numbers the extra term can be of the same order as, say, the production term. In such cases the rate of strain clearly is not small and calls the accuracy of the first-order model into question.

The choice of Reynolds number in compressible flow is critical, since differences amounting to a factor of 20 can arise at a Mach number of 5 depending on whether wall or freestream values of density and viscosity are specified. The evidence[3,13] suggests that low-Reynolds-number influence is located in the viscous superlayer, implying the use of freestream values. In these terms nozzle flows do not exhibit low Reynolds numbers. Mixing lengths predicted without dilatation and low-Reynolds-number corrections (Fig. 2) are high primarily because of the favorable pressure gradients. The effect of a low-Reynolds-number correction is to increase the results of Fig. 2 by a small factor whose magnitude depends upon the definition of Reynolds number employed.

Conclusions

The evidence of Coles' profiles for zero pressure gradient suggests that even in the absence of transitional effects a rise in mixing length will occur at low Reynolds numbers. The persistence of transitional effects (e.g., overshoot of the power-law exponent, reduced wake strength) results in a further increase. The effect of acceleration as such is to retard the growth of the boundary layer, and hence the wake component, and if there are no further complicating effects the mixing length remains high. We propose that the low values found by Bushnell et al. on supersonic nozzle walls are a result of a further complicating effect, the modification of turbulence structure by bulk dilatation in highly-accelerated compressible turbulent boundary layers.

Acknowledgment

We are grateful to Mr. Dennis Bushnell for comments on an early draft of this Note.

References

[1] Bushnell, D. M., Cary, A. J. Jr., and Holley, B. B., "Mixing Length in Low Reynolds Number Compressible Turbulent Boundary Layers," *AIAA Journal*, Vol. 13, Aug. 1975, pp. 1119-1121.

[2] Baker, R. J., and Launder, B. E., "The Turbulent Boundary Layer with Foreign Gas Injection. –1. Measurements in Zero Pressure Gradient," *International Journal of Heat and Mass Transfer*, Vol. 17, 1974, pp. 275-291.

[3] Murlis, J., "The Structure of a Turbulent Boundary Layer at Low Reynolds Number," Ph.D. Thesis, Dept. of Aeronautics, Imperial College, Univ. of London, 1975.

[4] Inman, P. N., "Transitional and Low Reynolds Number Turbulent Boundary Layers," Ph.D. Thesis (in preparation), Dept. of Aeronautics, Imperial College, Univ. of London, 1981.

[5] Bradshaw, P., "Two More Wind Tunnels Driven by Aerofoil-Type Centrifugal Blowers," Dept. of Aeronautics, Imperial College, Univ. of London, Aero. Rept. 72-10, Feb. 1972.

[6] Graham, J.M.R., "The Development of the Turbulent Boundary Layer Behind a Transition Strip," Aeronautical Research Council, London, 31,492 F.M. 4090, Oct. 1969.

[7] Herring, H. J. and Norbury, J. F., "Some Experiments on Equilibrium Turbulent Boundary Layers in Favourable Pressure Gradients," *Journal of Fluid Mechanics*, Vol. 27, 1967, pp. 541-549.

[8] Coles, D., "The Turbulent Boundary Layer in a Compressible Fluid, Appendix A, A Manual of Experimental Boundary Layer Practice for Low-Speed Flow," Rand Corp., Santa Monica, Calif., R-403-PR, Sept. 1962.

[9] Bradshaw, P., Mizner, G. A., and Unsworth, K., "Calculation of Compressible Turbulent Boundary Layers on Straight-Tapered Swept Wings," *AIAA Journal*, Vol. 14, March 1976, pp. 399-400.

[10] Bradshaw, P., "The Effect of Mean Compression or Dilatation on the Turbulence Structure of Supersonic Boundary Layers," *Journal of Fluid Mechanics*, Vol. 63, 1974, pp. 449-464.

[11] Lobb, R. K., Winkler, E. M., and Persh, J., "NOL Hypersonic Tunnel No. 4 Results VII: Experimental Investigation of Turbulent Boundary Layers in Hypersonic Flow," NOL NAVORD Rept. 3880, 1955.

[12] Winter, K. G. and Gaudet, L., "Turbulent-Boundary Layers at High Reynolds Numbers at Mach Numbers Between 0.2 and 2.8," Aeronautical Research Council, London, R&M 3712, 1973.

[13] Falco, R. E., "Coherent Motions in the Outer Region of Turbulent Boundary Layers," *The Physics of Fluids*, Vol. 20, Oct. 1977, pp. S124-S132.

AIAA 81-4110

Scaling of Interactions of Cylinders with Supersonic Turbulent Boundary Layers

David S. Dolling* and Seymour M. Bogdonoff†
Princeton University, Princeton, N.J.

Introduction

THE upstream extent of the disturbed flowfield induced by a cylindrical protuberance (or hemi-cylindrically blunted fin) in a supersonic turbulent boundary layer has been studied experimentally by many investigators.[1-13] In the past, protuberances have generally been classified as small or large[2] "on the basis of their height, h, relative to the 99% velocity boundary layer thickness δ." A protuberance is generally considered to be large if it produces the "asymptotic result," a condition occurring when further increases in h do not increase the extent of the disturbed flowfield. It has been noted[2] that "the asymptotic result is obtained rather quickly for $h>\delta$." In contrast, in a recent study[3] at a freestream Mach number M of 5.3, cylindrical protuberances of different diameters D were classified as being small if $h<D$ and large if $h>2D$. However, no physical explanation was offered for the latter result, nor mention made as to whether it should be interpreted as a general conclusion.

This apparent contradiction raises questions of which geometric and flow parameters are physically significant, and under what conditions is the asymptotic result obtained. These are the questions addressed by this Note. In addition, a simple approach is outlined for estimating if a cylindrical protuberance of given dimensions will generate the asymptotic result. In the following discussion, the incoming boundary layer is assumed turbulent. A laminar boundary layer does not change the basic approach, but does modify the quantitative result, as will be mentioned later.

Discussion

Physically, the asymptotic result occurs when the root shock structure is isolated from the free end by a two-dimensional central region in which the leading edge surface pressure Pw is equal to the freestream pitot value Pt_2. This condition is sketched in Fig. 1a. Typically, it occurs when h/htp is about 2-3, where htp is the asymptotic triple point height. Increasing h further only increases the extent of the central region, with no effect on the root region. Decreasing h, as shown in Fig. 1b, eliminates the central region, causing an interaction between the root and free-end flowfields. This, in turn, results in a reduction of asymptotic centerline upstream influence Lu.

Received Sept. 16, 1980; revision received Feb. 2, 1981.

*Professional Research Staff Member & Lecturer, Mechanical & Aerospace Engineering Dept. Member AIAA.

†Professor and Chairman, Mechanical & Aerospace Engineering Dept. Fellow AIAA.

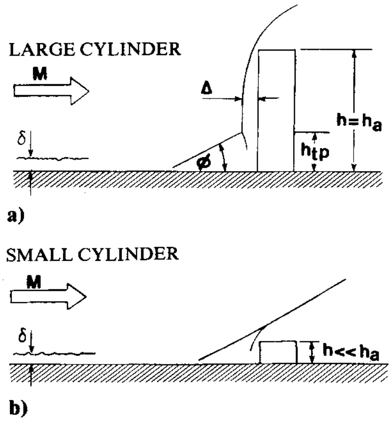

Fig. 1 Schematic of shock structure for a) "large" and b) "small" cylinders.

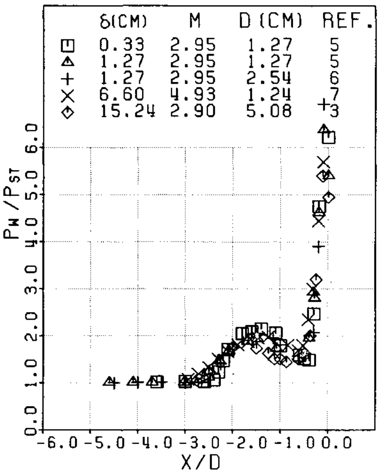

Fig. 2 Use of D as an upstream distance scaling parameter.

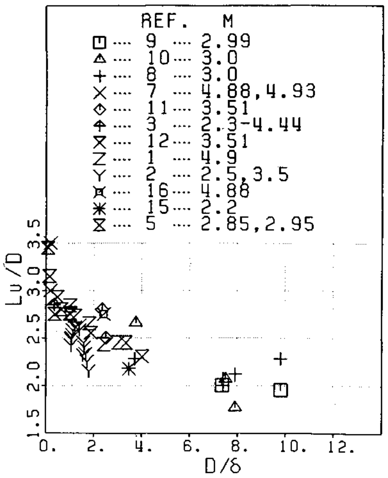

Fig. 3 Upstream influence as a function of D/δ.

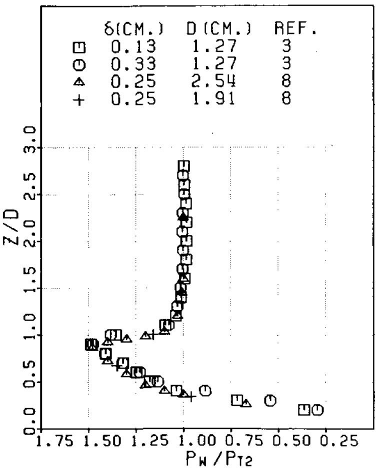

Fig. 4 Dependence of triple point height on D.

Following the approach of Westkaemper,[1] htp can be approximated by the expression

$$htp = (Lu - \Delta)\tan\phi \tag{1}$$

where Δ is the bowshock detachment distance and ϕ the mean angle of the centerline oblique shock wave. Westkaemper estimated ϕ using an expression developed by Truitt[14] for the pressure rise ΔP occurring in the interaction of a lambda footed bow shock wave and a turbulent boundary layer on a flat plate, namely

$$\frac{\Delta P}{q} = \frac{K}{(M \cdot Re)^{0.2}} \tag{2}$$

where q is the freestream dynamic pressure and Re the local Reynolds number. An empirical constant K was found by Truitt to be about 5.6. Knowing M and ΔP, oblique shock wave relations can be used to obtain ϕ. This technique was tested by the present authors on eight different experiments in which Schlieren photographs of the asymptotic root shock structure had been taken, and it was found that ϕ could be predicted to within ± 2 deg. Some uncertainty exists since a microsecond spark photograph "freezes" a shock structure which is known to be highly unsteady.[6]

The key parameter needed for estimating htp is the asymptotic centerline upstream influence Lu. Experiments carried out at Princeton, together with results from other studies, show that Lu depends primarily on D and only weakly on δ. Centerline pressure distributions, from the Princeton study[5,6] and others,[3,7] plotted in Fig. 2, illustrate this over a wide range of boundary layer thicknesses. In this figure, the wall pressures are normalized by the undisturbed freestream static value Pst. X, the distance along the centerline, is measured from the fin leading edge and is negative in the upstream direction.

The effect on Lu of changes in M is not entirely clear. Δ depends on M and D, but is typically a small proportion of Lu. In Ref. 2, tests were made over the range of $2.5 \le M \le 4.5$, and a trend of increasing Lu with increasing M was observed. On the other hand, measurements made over the range $2.3 \le M \le 4.44$, and reported on in Ref. 3, show no significant effect of M on Lu. This apparent contradiction may be due to the fact that in changing M, δ changes, which alters the ratio D/δ. It was observed in the Princeton study that, for fixed M and δ, Lu is primarily dependent on D, but also weakly dependent on D/δ. Other experiments exhibit a similar trend, as shown in Fig. 3. The data points on this figure were estimated from either surface oil patterns or pressure or heat transfer rate distributions. Using different techniques introduces some scatter, but does not mask the basic trend.

For given incoming flow conditions, the discussion above shows that htp, and therefore h_a (the height needed to generate the asymptotic result), depend primarily on D and to second order on D/δ. Measurements of leading edge pressures on different diameter fin models with different incoming boundary layers show this to be the case. Data from the Princeton study and from Ref. 8 are shown in Fig. 4 where Z is measured up the leading edge from the fin root. In these cases, in which $3.8 \le D/\delta \le 10$, the root shock wave structure is predominantly above the boundary layer edge. At lower values of D/δ, the root shock structure will be immersed in the boundary layer and htp increases (i.e., Fig. 17 of Ref. 5), but to first order it is still mainly dependent on D.

If h_a is taken as being $3htp$, then for the examples shown in Fig. 4, the required values of h_a would vary from 9 to 24δ.

However, in terms of D, the required h_a is $2.4D$ in all four cases. For a given δ, doubling D will approximately double both the absolute values of Lu and htp. In turn, this will double the required value of h_a, but the ratio h_a/D will remain the same. The main effect of δ lies in its second-order influence on Lu.

h_a will also depend on the protuberance shape. For fixed flow conditions, experiments at Princeton show that Lu for a flat faced fin of thickness t is approximately twice that for a hemi-cylindrically blunted fin with $D=t$. Since ϕ stays the same, both htp and the required h_a for the former are about twice those of the latter. For a given value of δ, h_a/δ will differ by a factor of 2 for the two cases, whereas h_a/htp will be about the same.

The ideas above explain several apparent anomalies existing in the literature. For example, Waltrup et al.,[15] observed the asymptotic result at an h/δ of about 10, whereas Price and Stallings,[3] who tested five sweptback fin models in the range $0.17 \leq h/\delta \leq 1.07$ found that it had already occurred at the lowest value of h/δ. Waltrup's flow conditions input to Eqs. (1) and (2) gave $htp = 1.9$ cm (or 4.3δ), suggesting that the asymptotic result would occur at 2-3 times this height (i.e., around 9-13δ), which was the case. For Price and Stallings' flow, the equations give $htp \simeq 0.07\delta$ so that it was not surprising that even at an h/δ of 0.17 the asymptotic result had already occurred.

In the fully laminar case, recent measurements by Hung and Clauss[13] show that Lu also scales with D, and typically is between 9 and $12D$. In addition, the exponents of M and Re and the constant K in Truitt's relation [i.e., Eq. (2)] will differ from those for turbulent flow.

Concluding Remarks

The ratio h/δ is not the physically relevant parameter when determining if a given protuberance will generate the asymptotic flowfield. Physically, the latter occurs when the fin height h is of order 2-3 htp and, since for a given flow htp depends primarily on D, the important parameter is h/D. A unique value of h/D, valid for arbitrary conditions, cannot be specified since htp depends also on M and Re.

Acknowledgments

This work was sponsored by the U.S. Naval Air Systems Command, Contract N60921-78-C-0068, and the U.S. Army Research Office, Grant DAAG 29-77-G-0234.

References

[1] Westkaemper, J. C., "Turbulent Boundary Layer Separation Ahead of Cylinders," *AIAA Journal*, Vol. 6, July 1968, pp. 1352-1355.

[2] Sedney, R. and Kitchens, C. W. Jr., "Separation Ahead of Protuberances in Supersonic Turbulent Boundary Layers," *AIAA Journal*, Vol. 15, April 1977, pp. 546-552.

[3] Price, A. E. and Stallings, R. L., "Investigation of Turbulent Separated Flows in the Vicinity of Fin Type Protuberances at Supersonic Mach Numbers," NASA TN D-3804, Feb. 1967.

[4] Dolling, D. S., Cosad, C. D., and Bogdonoff, S. M., "Three-Dimensional Shock Wave Turbulent Boundary Layer Interactions—A Parametric Study of Blunt Fin-Induced Flows," AIAA Paper 78-159, Huntsville, Ala., Jan. 1978.

[5] Dolling, D. S., Cosad, C. D., and Bogdonoff, S. M., "An Examination of Blunt Fin-Induced Shock Wave Turbulent Boundary Layer Interactions," AIAA Paper 79-0068, New Orleans, La., Jan. 1979.

[6] Dolling, D. S. and Bogdonoff, S. M., "Experimental Investigation of Three-Dimensional Shock Wave Turbulent Boundary Layer Interaction—An Exploratory Study of Blunt Fin Induced Flows," Princeton Univ. M.A.E. Rept. 1468, March 1980.

[7] Winkelmann, A. E., "Experimental Investigations of a Fin Protuberance Partially Immersed in a Turbulent Boundary Layer at Mach 5," NOLTR-72-33, Jan. 1972.

[8] Kaufman, L. G., Korkegi, R. H., and Morton, L. C., "Shock Impingement Caused by Boundary Layer Separation Ahead of Blunt Fins," ARL 72-0118, Aug. 1972.

[9] Lucas, E. J., "Investigation of Blunt Fin-Induced Flow Separation Region on a Flat Plate at Mach Numbers 2.5 to 4.0," AEDC-TR-70-265, Jan. 1971.

[10] Uselton, J. C., "Fin Shock-Boundary Layer Interaction Tests on a Flat Plate with Blunted Fins at $M=3$ and 5," AEDC-TR-67-113, June 1967.

[11] Burbank, P. B. and Strass, H. K., "Heat Transfer to Surface and Protuberances in a Supersonic Turbulent Boundary Layer," NACA RM L58E01a, July 14, 1958.

[12] Burbank, P. B., Newlander, R. A., and Collins, I. K., "Heat-Transfer and Pressure Measurements on a Flat-Plate Surface and Heat Transfer Measurements on Attached Protuberances in a Supersonic Turbulent Boundary Layer at Mach Numbers of 2.65, 3.51, and 4.44," NASA TN D-1372, Dec. 1962.

[13] Hung, F. T. and Clauss, J. M., "Three-Dimensional Protuberance Interference Heating in High Speed Flow," AIAA Paper 80-0289, Pasadena, Calif., Jan. 1980.

[14] Truitt, R. W., "Hypersonic Turbulent Boundary-Layer Interference Heat Transfer in Vicinity of Protuberances," *AIAA Journal*, Vol. 3, Sept. 1965, pp. 1754-1755.

[15] Waltrup, P. J., Hall, D.G., and Schetz, J. A., "Flowfield in the Vicinity of Cylindrical Protuberances on a Flat Plate in Supersonic Flow," *Journal of Spacecraft and Rockets*, Vol. 5, Jan. 1968, pp. 127-128.

[16] Miller, W. H., "Pressure Distributions on Single and Tandem Cylinders Mounted on a Flat Plate in Mach Number 5.0 Flow," Univ. of Texas, Defense Research Laboratory, DRL 538, 1 June 1966.

AIAA 81-4111

Supercritical Swirling Flows in Convergent Nozzles

P. W. Carpenter*
University of Exeter, Exeter, England

Introduction

It has generally been assumed that the topic discussed in this Note has already been more than adequately covered in numerous papers (fairly comprehensive lists of references are given in Refs. 1 and 2). Previous authors, however, have apparently overlooked the fact that the quasicylindrical theory for compressible swirling flow in convergent-divergent nozzles leads to a paradox when applied to convergent nozzles. Accordingly, the present Note sets out to describe this paradox and to present a simple theory for supercritical swirling flow in convergent nozzles. Some results for mass flux and thrust coefficients are also given.

For nonswirling supercritical flows it is generally assumed that one-dimensional theory is equally applicable to convergent and convergent-divergent nozzles. The theory is found to give good results when the area change is gradual and the nozzle wall curvature small. It has also been generally accepted, at least implicitly, that the quasicylindrical theory for swirling compressible flow is equally applicable to both types of nozzle. There has been considerable difficulty, though, in defining a suitable choking criterion for swirling nozzle flows. Many previous investigators[3-5] have used the concept of maximum-mass-flux to obtain a choking criterion. Norton et al.[6] and Carpenter and Johannesen,[1,2] however, made no additional assumptions about choking beyond those usually made for quasicylindrical theory. The latter showed that the maximum-mass-flux criterion is not strictly valid but gave reasonable results for most swirling flows in convergent-divergent nozzles. They also showed that, owing to the

Received Sept 23, 1980.

*Lecturer, Dept. of Engineering Science. Member AIAA.